SM 3H

8.4 Multiple Angle Identities and Equations 2017-18

Name _____ Date _____ Period _____

Use the appropriate <u>sum or difference identity</u> to prove the double-angle identity. Show work! Remember you <u>cannot</u> use the identity you are proving in the proof of the identity.

1. $\cos 2u = \cos^2 u - \sin^2 u$

2. $\cos 2u = 1 - 2\sin^2 u$

Find the exact value of each expression using <u>double-angle identities</u>. Show work (do not use the unit circle).

3. $\sin(90°)$

4. $\tan(60°)$

5. $\cos\left(\frac{4\pi}{3}\right)$

6. $\sin\left(\frac{2\pi}{3}\right)$

Use identities to simplify each expression. Do not use a calculator.

7. $2\sin(13°)\cos(13°)$

8. $2\cos^2(22.5°) - 1$

9. $\frac{2\tan(15°)}{1-\tan^2(15°)}$

10. $\cos^2\left(\frac{\pi}{9}\right) - \sin^2\left(\frac{\pi}{9}\right)$

Find the exact value of each expression using half-angle identities.

11. $\sin(15°)$

12. $\tan(15°)$

13. $\cos\left(\frac{\pi}{8}\right)$

14. $\sin\left(22.5°\right)$

For each equation determine whether the positive or negative sign makes the equation correct. Do not use a calculator.

15. $\sin\left(118.5°\right)=\pm\sqrt{\frac{1-\cos\left(237°\right)}{2}}$

16. $\cos\left(100°\right)=\pm\sqrt{\frac{1+\cos\left(200°\right)}{2}}$

17. $\cos\left(\frac{9\pi}{7}\right)=\pm\sqrt{\frac{1+\cos\left(\frac{18\pi}{7}\right)}{2}}$

18. $\tan\left(\frac{17\pi}{12}\right)=\pm\sqrt{\frac{1-\cos\left(\frac{17\pi}{6}\right)}{1+\cos\left(\frac{17\pi}{6}\right)}}$

Find <u>all</u> <u>real</u> <u>numbers</u> that satisfy the equation. Write answers in terms of π.

19. $\cos\left(\frac{x}{2}\right)=\frac{1}{2}$

20. $\cos\left(3x\right)=1$

21. $\tan\left(4x\right)=0$

22. $2\sin\left(2x\right)=-\sqrt{2}$

Find all values of α in [0°, 360°) that satisfy each equation.

23. $2\sin(2x)+\sqrt{3}=0$

24. $2\cos(2x)+1=0$

25. $\sqrt{2}\cos(2x)-1=0$

26. $2\sin(2x)-1=0$

Find all real numbers in the interval [0, 2π) that satisfy each equation.

27. $2\cos(2x)-1=0$

28. $\tan(3x)-1=0$

29. $\sqrt{3}\tan\left(\frac{x}{2}\right)-1=0$

30. $\sqrt{2}\sin\left(\frac{x}{3}\right)-1=0$

31. $\sin 2x=2\sin x$

32. $\cos 2x=\sin x$

33. $\cos 2x + \cos x = 0$

34. The vertical position of a floating ball in an experimental wave tank is given by the equation $x = 2\sin\left(\frac{\pi}{3}t\right)$, where *x* is the number of feet above sea level and *t* is the time in seconds. For what values of *t* is the ball $\sqrt{3}$ ft above sea level?

Prove that each equation is an identity.

35. $\cos^4 x - \sin^4 x = \cos 2x$

36. $(\sin x - \cos x)^2 = 1 - \sin 2x$

37. $\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$

38. $2\sin^2\left(\frac{x}{2}\right) = \frac{\sin^2 x}{1 + \cos x}$

39. $\sin 4u = 2\sin 2u \cos 2u$

40. $2\csc 2x = \csc^2 x \tan x$

In each case, find $\sin\alpha, \cos\alpha, \tan\alpha, \csc\alpha, \sec\alpha,$ *and* $\cot\alpha$.

41. $\cos 2\alpha = \frac{3}{5}$ and $0° < 2\alpha < 90°$

Solve each problem.

42. Find the exact value of $\sin(2\alpha)$ given that $\sin(\alpha) = \frac{3}{5}$ and α is in quadrant II.

43. Find the exact value of $\cos(2\alpha)$ given that $\sin(\alpha) = \frac{8}{17}$ and α is in quadrant II.